DARTS workshop

Reservoir simulation: few important explanations

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Modern reservoir simulation

- Highly-implicit time approximation
- Complex unstructured grid models
  - Resolution down to geological model
  - Complex features including fractures
- Simulation of ensemble of models
  - Uncertainty quantification
  - Data assimilation and optimization
- Complex wells and controls
  - Multi-segmented well models

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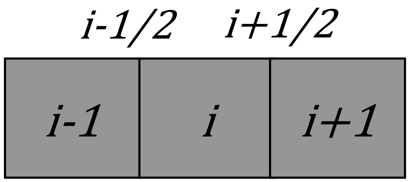
Single phase equation

$$\frac{\partial \phi \rho}{\partial t} + \nabla \rho \mathbf{U} + \rho q = 0$$

$$\phi = \phi(p), \rho = \rho(p), T = T(p), q = q(p)$$

Finite-volume discretization



$$\frac{(\phi\rho)_i{}^{n+1} - (\phi\rho)_i{}^{n}}{\Delta t} + \frac{(\rho U)_{i+1/2} - (\rho U)_{i-1/2}}{\Delta x} + (\rho q)_i = 0$$

Fully implicit approximation

$$R_i = V_i \frac{(\phi\rho)_i{}^{n+1} - (\phi\rho)_i{}^{n}}{\Delta t} - (\rho T)_{i+\frac{1}{2}}(p_{i+1} - p_i)^{n+1} + (\rho T)_{i-1/2}(p_i - p_{i-1})^{n+1} + (\rho q)_i{}^{n+1}$$

$$R_i(p_{i-1}{}^{n+1}, p_i{}^{n+1}, p_{i+1}{}^{n+1}) = 0$$

Nonlinear solution

Need to solve: $\mathbf{R}(\mathbf{x}) = \mathbf{0}, \quad \mathbf{x} = \mathbf{p}|_{t=t^{n+1}}; \quad \mathbf{x}_{\boldsymbol{n}} = \mathbf{p}|_{t=t^n}$

Newton-Raphson (iterative) method:

Linear iterations

Nonlinear iterations

$$\mathbf{x}^{\mathbf{0}} = \mathbf{x}_{\boldsymbol{n}}; \;\; \mathbf{J}(\mathbf{x}^0)(\mathbf{x}^1 - \mathbf{x}^0) = -\mathbf{R}(\mathbf{x}^0) \quad \Longrightarrow \quad \mathbf{A}_0\mathbf{y}_0 = \mathbf{b}_0$$

$$\mathbf{x}^1 = \mathbf{x}^0 + \mathbf{y}_0; \;\; \mathbf{J}(\mathbf{x}^1)(\mathbf{x}^2 - \mathbf{x}^1) = -\mathbf{R}(\mathbf{x}^1) \quad \Longrightarrow \quad \mathbf{A}_1\mathbf{y}_1 = \mathbf{b}_1$$

$$\dots$$

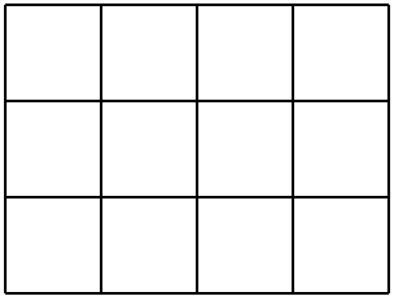
$$\left\|\mathbf{R}(\mathbf{x}^k)\right\| < \varepsilon_1 \;\; \& \; \|\mathbf{y}_k\| < \varepsilon_2$$

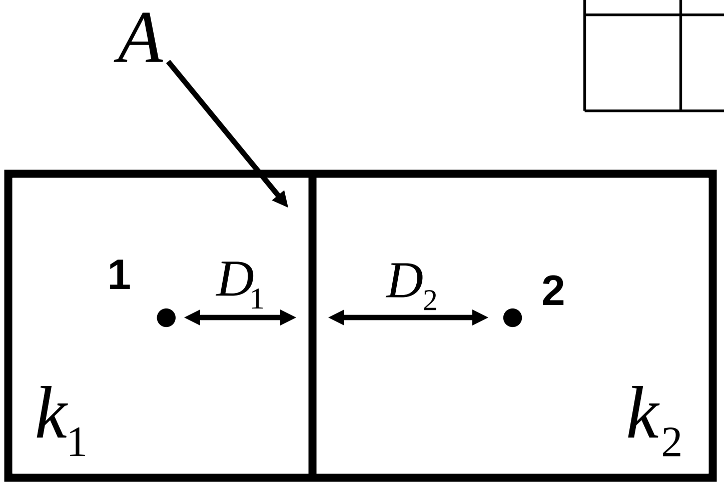
$$\boldsymbol{p}^{n+1} = \mathbf{x}^k \;\; \Longrightarrow t = t + \Delta t, \;\; \boldsymbol{p}^n = \boldsymbol{p}^{n+1}$$

start new timestep

Two-Point Flux Approximation

Cartesian grid





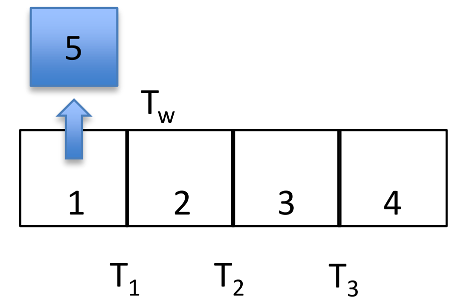
$$Q_{12} = T_{12}\left(P_1 - P_2\right)$$

$$T_{12} = \Upsilon_{12}\lambda$$

$$\Upsilon_{12} = \left(\frac{\alpha_1\alpha_2}{\alpha_1 + \alpha_2}\right)$$

$$\alpha_i = A_i\frac{k_i}{D_i}$$

Connection list: residual assemble



1) Connection list represents model connectivity

$$\{1{,}2{:}T_1\}, \{2{,}3{:}T_2\}, \{3{,}4{:}T_3\}, \{1{,}5{:}T_w\}$$

2) Flow rate between two adjacent control volumes

$$Q_k = T_k\left(p_i - p_j\right), \quad Q_w = T_w(p_i - p_w)$$

3) Mass conservation in residual form for each control volume

- Loop over control volumes to account for accumulation term defined as $A_i = V_i(\phi\rho)_i$

$$i: \; R_i = {A_i}^{n+1} - {A_i}^{n}$$

- Loop over connections to account for in/outflow between blocks

$$k \to \{i,j\} : R_i = R_i + \Delta t \; {Q_k}^{n+1}, R_j = R_j - \Delta t \; {Q_k}^{n+1}$$

- Loop over wells to account for source/sink terms

$$q \to \{i,w\} : R_i = R_i + \Delta t \; {Q_w}^{n+1}, R_w = R_w - \Delta t \; {Q_w}^{n+1}$$

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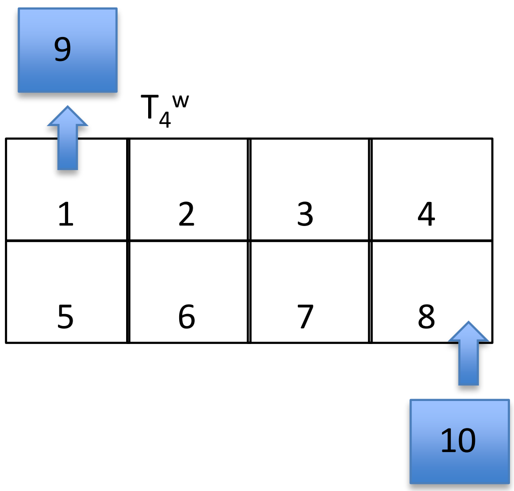
Simulation in 2D



T_1: 1,2
T_2: 2,3
T_3: 3,4
T_4: 5,6
T_5: 6,7
T_6: 7,8
T_7: 1,5
T_8: 2,6
T_9: 3,7
T_{10}: 4,8
T_{11}: 9,1
T_{12}: 10,8

	1	2	3	4	5	6	7	8	9	10
1	x	x			x				x	
2	x	x	x			x				
3		x	x	x			x			
4			x	x				x		
5	x				x	x				
6		x			x	x	x			
7			x			x	x	x		
8				x			x	x		x
9	x								x	
10								x		x

x - non-zero elements

Two-phase nonlinear system

$$\mathbf{A}_i(p_i, S_i) = (\phi V)_i \begin{bmatrix} \rho_o S_o \\ \rho_w S_w \end{bmatrix}_i$$

$$\mathbf{U}_p = -\mathbf{k}\frac{k_{rp}}{\mu_p}\nabla p_p \Longrightarrow Q_k(\mathbf{p}, \mathbf{S}) = \begin{bmatrix} T_{o,k}\left(p_{o,i} - p_{o,j}\right) \\ T_{w,k}\left(p_{w,i} - p_{w,j}\right) \end{bmatrix}$$

$$S_o + S_w = 1, \qquad p_o - p_w = P_{cow}(S_w)$$

Solve nonlinear equation: $\mathbf{R}(\mathbf{x}) = \mathbf{0}$

$$\mathbf{x}_i = [p_o, S_w]^T{}_i\,;\; S_w = 1 - S_o;\; p_w = P_{cow}(S_w) - p_o$$

Geothermal model

$$\frac{\partial}{\partial t}\big(\phi(S_w\rho_w + S_s\rho_s)\big) + \nabla(\rho_w \mathbf{U}_w + \rho_s \mathbf{U}_s) = 0$$

$$\frac{\partial}{\partial t}(\phi(u_w S_w \rho_w + u_s S_s \rho_s) + (1-\phi)u_r) + \nabla\big(h_w\rho_w\mathbf{U}_w + h_s\rho_s\mathbf{U}_s + \phi(\mathbf{G}_w + \mathbf{G}_g) + (1-\phi)\mathbf{G}_r\big) = 0$$

$\mathbf{U}_\mathrm{p}$ – phase flux (Darcy assumptions)

$\mathbf{G}_\mathrm{p}$ – conduction flux (homogeneous temperature)

$p = \mathrm{w,s}$ (water or steam)

$\mathrm{S_p}$ – phase saturation

ρ_p – phase density

ϕ – porosity

$\mathrm{u_p}$ – phase energy

$\mathrm{h_p}$ – phase enthalpy

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Geothermal system

$$A_i(\boldsymbol{\omega}) = (V)_i \begin{bmatrix} \phi(S_w\rho_w + S_s\rho_s) \\ \phi(u_w\rho_w + u_s\rho_s) + (1-\phi)u_r \end{bmatrix}_i,$$

$$Q_k(\boldsymbol{\omega}) = \begin{bmatrix} (T_{w,k}+T_{s,k})\left(p_i - p_j\right) \\ (h_w T_{w,k} + h_s T_{s,k})\left(p_i - p_j\right) \end{bmatrix}$$

$$G_k(\boldsymbol{\omega}) = -\theta_\kappa\left(T_i - T_j\right)$$

$$\theta_\kappa = \mathcal{H}[\phi(S_w\kappa_w + S_s\kappa_s) + (1-\phi)\boldsymbol{\kappa_r}]$$

Solve nonlinear equation: $\mathbf{R}(\boldsymbol{\omega}) = \mathbf{0}$

Two-phase geothermal

$$F_w = f_w(p, T) - f_s(p, T) = 0$$

- Natural variables formulation:
  - Nonlinear unknowns: p, T, S_w
  - Solve conservation equations + equilibrium fully coupled
- Molar variables formulation:
  - Nonlinear unknowns: p, h
  - Solve (3) on each global iteration
  - Find derivatives using inverse theorem: $\mathbf{\frac{dx}{dz} = \frac{dF}{dz}\left(\frac{dF}{dx}\right)^{-1}}$

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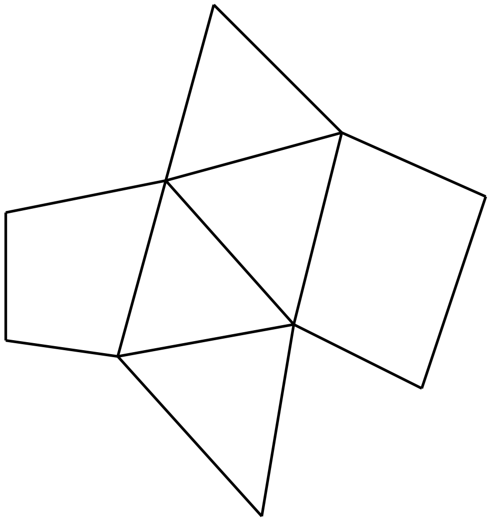
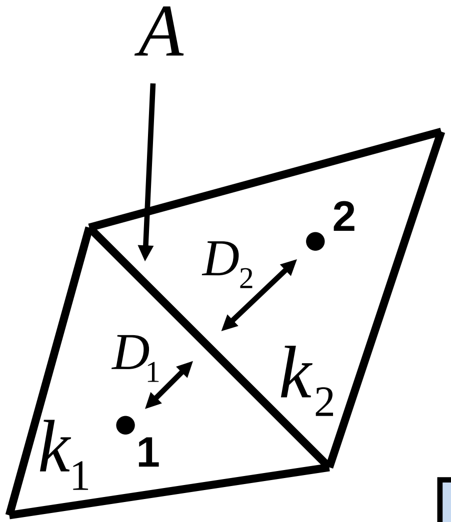
Transmissibility (unstructured)

Unstructured grid





$$T_{12} = \Upsilon_{12}\lambda$$

$$\Upsilon_{12} = \left(\frac{\alpha_1\alpha_2}{\alpha_1 + \alpha_2}\right)$$

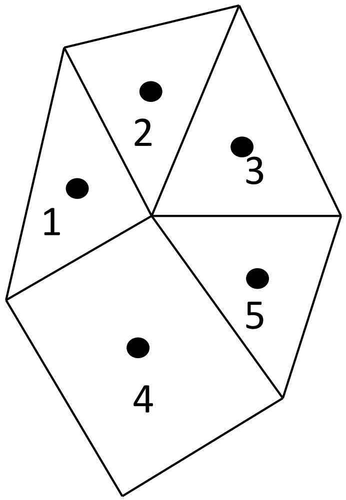
$$\alpha_i = A_i \frac{k_i}{D_i}$$

$$Q_{12} = T_{12}\left(P_1 - P_2\right)$$

Unstructured grid

1) Connection list represents model connectivity

$\{1,2:T_1\}, \{1,4:T_2\}, \{2,3:T_3\}, \{3,5:T_4\}, \{4,5:T_5\}$



2) Flow rate between two adjacent control volumes

$$Q_k = T_k\left(p_i - p_j\right)$$

3) Mass conservation for each control volume

- Loop over control volumes to account for accumulation and source term

$$i: \; R_i = A_i^{\;n+1} - A_i^{\;n}$$

- Loop over connections to account for in/outflow

$$k \rightarrow \{i,j\} : R_i = R_i + \Delta t \, Q_k^{\;n+1}, R_j = R_j - \Delta t \, Q_k^{\;n+1}$$

Solve nonlinear equation: $\mathbf{R(p) = 0}$

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Chemical reactions

Component mass balance:

$$\frac{\partial}{\partial t}\left(\phi \sum_{p} \rho_p S_p x_{cp}\right) + \nabla \cdot F_c + q_{w,c} = \sum_{r=1}^{n_r} v_{c,r} r_r$$

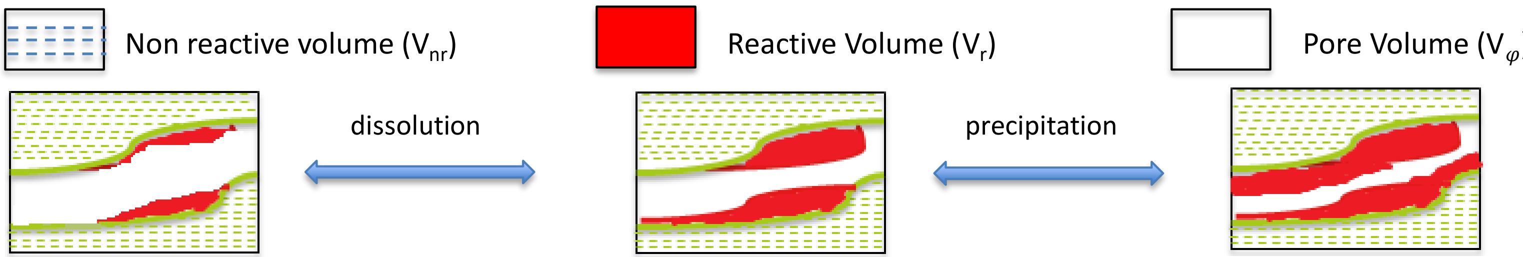
$$\boldsymbol{a_c} + \boldsymbol{l_c} = \boldsymbol{Vr}$$

Chemical reactions:

$$K_{sp} - Q_{sp} = 0 \qquad \text{equilibrium}$$

$$a_c^k + l_c^k = \boldsymbol{vr}^k \qquad \text{kinetic}$$

Porosity-permeability treatment



- Bulk volume is a sum of reactive (V_r), non reactive (V_{nr}) and pore volume (V_φ).

$$V_b = V_\varphi + V_r + V_{nr}$$

- We introduce the concept of total porosity which is defined as

$$\varphi^T = \varphi_r + \varphi$$

- Fluid porosity can be calculated using the mineral saturation values

$$\varphi = \varphi^T \left(1 - \sum_i^{n_m} S_i^{n+1}\right)$$

- Power law transmissibility multiplier using upstream fluid porosity

$$T^M(\varphi) = \left(\frac{\varphi}{\varphi^T}\right)^A = \left(1 - \sum_i^{n_m} S_i^{n+1}\right)^A$$

In this way, $T^M \in [0, 1]$, i.e. at any time: $T = \Gamma^{max}\, T^M$